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Film-Condensation Process Controlled by a Darcy Cooling Fluid Flow

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The film-condensation process of a saturated vapor in contact with the external surfaces of a porous channel, caused by a Darcy cooling fluid flow, is studied. The longitudinal heat conduction effects in the walls of the channel are included. The momentum and energy balance equations are reduced to a system of integro–differential equations with five nondimensional parameters: the Prandtl number of the condensed fluid Pr_c, the Jacob number Ja, a nondimensional plate thermal conductivity α, and the aspect ratio of the walls ε and β defined by the ratio of the thermal resistance of the condensed layer to the thermal resistance of the Darcy cooling flow. The resulting governing equations are integrated in the asymptotic limit $Ja \longrightarrow 0$ to obtain the spatial evolution of the condensed-layer thickness and temperature of the walls as a function of the longitudinal coordinate. For practical values of the parameters α and β, the present analysis shows that both effects modify the well-known Nusselt solution for an isothermal wall.

Nomenclature

c	=	specific heat of the cooling fluid
c_c	=	specific heat of the condensed phase
f_c	=	nondimensional stream function introduced in Eq. (10)
g	=	acceleration of gravity
h	=	thickness of the plate
h_{fg}	=	latent heat of condensation
Ja	=	Jacob number defined in Eq. (2)
L	=	length of the wall
m'	=	mass flow rate of condensed fluid
Nu_c	=	Nusselt number defined in Eq. (26)
Pr	=	Prandtl number of the cooling flow
Pr_c	=	Prandtl number of the condensed fluid
Re	=	Reynolds number of the cooling flow
Re_c	=	Reynolds number of the condensed fluid
T	=	temperature
T_s	=	temperature of the saturated vapor
T_∞	=	freestream temperature of the cooling fluid flow
U	=	uniform Darcy velocity of the cooling fluid flow
u, v	=	nondimensional longitudinal and transverse velocities
u_c	=	characteristic longitudinal velocity of the condensed fluid
$\overline{u}, \overline{v}$	=	longitudinal and transverse velocities in physical units
x, y	=	Cartesian coordinates
α	=	heat conduction parameter defined in Eqs. (7)
β	=	nondimensional parameter defined in Eqs. (7)
Δ	=	normalized thickness of the condensed layer
δ	=	thermal thickness of the cooling flow
δ_c	=	thickness of the condensed layer
δ_{cL}	=	thickness of the condensed layer at $\chi = 1$
ε	=	aspect ratio of the plate
ζ	=	nondimensional inner coordinate defined in Eq. (9)
η_c	=	nondimensional transversal coordinate for the condensed fluid flow
θ	=	nondimensional temperature of the cooling fluid
θ_c	=	nondimensional temperature of the condensed layer
θ_w	=	nondimensional temperature of the wall
λ	=	thermal conductivity of the cooling fluid
λ_c	=	thermal conductivity of the condensed phase
λ_w	=	plate thermal conductivity
μ_c	=	dynamic viscosity of the condensed fluid
ν	=	kinematic coefficient of viscosity of the cooling fluid
ν_c	=	kinematic coefficient of viscosity of the condensed fluid
ρ	=	Darcy flow density
ρ_c	=	condensed fluid density
σ	=	stretched inner coordinate defined in Eq. (46)
χ	=	nondimensional longitudinal coordinate defined in Eq. (9)
ψ	=	nondimensional thickness of the condensed layer defined in Eq. (46)

Subscripts

c	=	condensed fluid
e	=	$\alpha = 0$
L	=	conditions at trailing lower edge of the wall
l	=	conditions at leading upper edge of the wall
w	=	conditions at wall

I. Introduction

SINCE the pioneering paper of Nusselt,[1] the laminar-film condensation on surfaces with the inclusion of dynamic and thermal effects has been widely studied in the specialized literature. In general, these effects are related to the presence of shear stresses at the interface between the condensed and vapor phases, saturated and noncondensable vapor mixtures, natural and forced regimes, gravity forces, superheated vapors, waving movement, and transition to turbulence, among others. During the past decades, new analytical treatments of this process have been reexamined to improve the theoretical predictions provide by Nusselt's simple theory. However, in these studies only uniform thermal conditions at the exposed surfaces to the condensation process are considered. The state of the art of the film-condensation process using uniform thermal boundary conditions can be found by Rose[2] and more recently by Tanasawa,[3] where different theoretical and experimental analyses show the most significative contributions related to this class of problems.

In contrast with the foregoing studies, Patankar and Sparrow[4] were the first to solve numerically the laminar-film-condensation

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process on a cooled nonisothermal vertical fin or cylinder. Here, the condensation process is coupled to heat conduction within the fin, and using a similarity analysis, the authors concluded that the calculated fin heat transfer is lower than that corresponding to the isothermal-fin model. Later, it was shown by Wilkins[5] that an explicit solution is possible for the formulation of Patankar and Sparrow. The last contributions show that the studies of film condensation on nonisothermal extended surfaces form a class by themselves and that the estimation of surface area requirements of the condenser by using classical Nusselt analysis, is not appropriate. To extend this new kind of nonisothermal case, Sarma et al.[6] studied the condensation process on a vertical plate fin of variable thickness. Brouwers[7] developed an analytical study for the condensation of a pure saturated vapor on a cooled channel plate, including the thermal interaction between the cooling liquid, the condensate, and the vapor. Analyzing different cases for co-, counter-, and cross-current condensation, the governing equations are solved numerically and the most relevant condensation heat transfer parameters are evaluated, showing the clear influence of this thermal interaction. Méndez and Treviño[8] solved the problem of film condensation on a side of a thin vertical surface caused by a forced cooling fluid that is flowing on the other side of the vertical plate. In Ref. 8, the effect of the longitudinal heat conduction through the wall drastically modifies the classical Nusselt solution. Mosaad[9] studied the laminar film-condensation process coupled across an impermeable vertical wall with a natural convection flow in a porous medium. He reported that the overall heat transfer coefficient is strongly influenced by the parameters associated with the natural convection flow.

In an effort to obtain solutions for nonisothermal conditions, in this paper the laminar-film condensation on the external sides of a porous vertical channel is analyzed. At the inner region of the porous channel, we assume a uniform Darcy cooling flow characterized by a temperature lower than the saturated temperature of the vapor. Therefore, the thermal coupling between the cooling convection flow, the walls of the vertical channel, and the condensed phase appears to offer a new theoretical extension to the fundamental work in laminar-film condensation. In this work, using regular and singular perturbations methods and the boundary-layer approximation for the cooling forced and condensed flows, it is shown that the film-condensation process and longitudinal heat conduction through the wall depends on, at least, five nondimensional parameters: the Prandtl number of the condensed fluid, Pr_c, the Jacob number Ja, and the nondimensional parameters α, ε, and β. The parameter α represents the competition between the heat flux conducted through the wall with the heat flux carried to the wall from the condensed phase, ε is the aspect ratio of the walls, and β defines the ratio of the thermal resistance of the condensed fluid to the thermal resistance of the forced cooling flow. We develop an asymptotic analysis for large and small values of α compared with unity to predict the spatial evolution of wall temperature and thickness of the condensate. Finally, we compare analytical solutions with the results obtained using numerical techniques.

II. Formulation and Order of Magnitude Analysis

The physical model under study is shown in Fig. 1. We consider a homogeneous porous channel bounded by two thin vertical heat conducting walls a distance H apart, length L, and thickness h, placed in a stagnant atmosphere filled with saturated vapor at a temperature T_s. For simplicity, both ends of the walls are connected with adiabatic surfaces. A downward Darcy cooling flow regime, with temperature $T_\infty < T_s$ and a uniform seepage velocity U, is imposed between both walls, thus, generating a thermal entrance region between both walls and a heat flux from the saturated vapor. This conjugated heat transfer process creates thin condensed films on the external surfaces of the walls. The condensed layers develop with increasing thicknesses in the direction of gravity. The density of the condensed fluid, ρ_c, is assumed to be constant and much larger than the vapor density ρ_v. The Cartesian coordinate system is located at the vertical symmetry plane, whose y axis points in the direction normal to the walls and its x axis points down in the longitudinal direction. Because of the geometrical and physical symmetries of

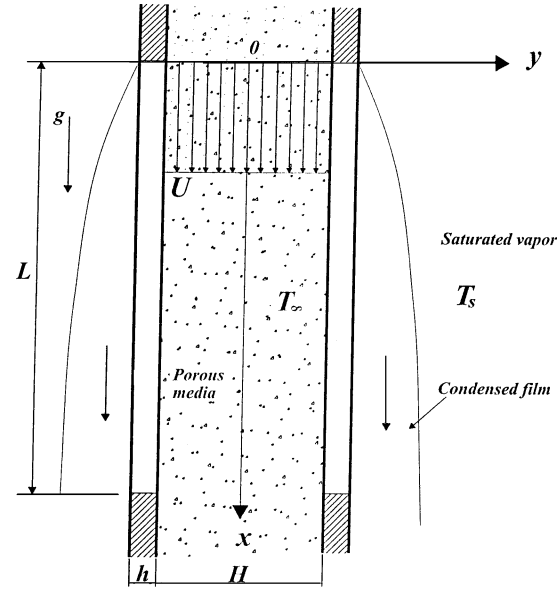

Fig. 1 Schematic diagram of studied physical model.

the model, we formulate the governing equations only for positive values of the y coordinate.

When classical Nusselt assumptions for the condensed fluid are used, the characteristic velocity and the condensed mass flow and the mass production rate, both by unit length, are of order

$$u_c \sim \frac{g}{\nu_c}\delta_c^2, \qquad m' \sim \frac{\rho_c g}{\nu_c}\delta_c^3, \qquad \frac{\mathrm{d}m'}{\mathrm{d}x} \sim \frac{\lambda_c \Delta T_c}{\delta_c h_{fg}} \tag{1}$$

respectively, where ΔT_c is the characteristic temperature difference in the condensed fluid. From these relationships, the global thickness of the condensed layer related to the length of the wall is given by

$$\delta_c/L \sim [(Ja/\gamma Pr_c)(\Delta T_c/\Delta T)]^{\frac{1}{4}}$$

$$\text{with} \qquad Ja = c_c \Delta T / h_{fg}, \qquad \gamma = gL^3 \big/ \nu_c^2 \tag{2}$$

where the Jacob number Ja is the ratio of the sensible energy absorbed by the liquid to the latent heat of the liquid during the condensation process and ΔT is the global temperature difference, $\Delta T = T_s - T_\infty$. In general, the Jacob number Ja and the parameter γ are very small and large compared with unity, respectively, and the Prandtl number Pr_c is of order unity[10]; thus, the limit $Ja/\gamma \to 0$ is fully justified if $(\Delta T_c/\Delta T) \sim 1$. Therefore, the ratio $\Delta T_c/\Delta T$ must be determined as part of the order-magnitude analysis. On the other hand, the Darcy flow can be analyzed by introducing a thermal boundary layer for high values of the Péclet number, $Pe = RePr$. It can be easily shown that the corresponding thickness of the thermal boundary layer related to the length of the wall is given by

$$\delta/L \sim [(H/L)(1/Pe)]^{\frac{1}{2}} \tag{3}$$

This estimate follows from the order-of-magnitude balance of vertical heat convection and horizontal heat conduction across the thermal boundary layer, that is,

$$\rho c U \Delta T_\infty / L \sim \lambda \Delta T_\infty \big/ \delta^2 \tag{4}$$

where ρ, c, and λ are the density, specific heat, and effective thermal conductivity of the Darcy fluid flow, respectively, and ΔT_∞ is the characteristic temperature difference across the Darcy flow.

The global temperature drop ΔT from the condensed fluid to the cooling flow is related to each temperature drop of the system as $\Delta T = \Delta T_c + \Delta T_w + \Delta T_\infty$, where ΔT_w is the characteristic temperature difference across the wall. By equating the heat fluxes in both surfaces of the wall, and using relationships (1–3), we can easily obtain that

$$\Delta T_c/\Delta T \sim (\alpha/\varepsilon^2)^{\frac{4}{3}}(\Delta T_w/\Delta T)^{\frac{4}{3}}$$
$$\Delta T_\infty/\Delta T \sim (\alpha/\varepsilon^2\beta)(\Delta T_w/\Delta T) \tag{5}$$

and using the preceding temperature drop ΔT, together with relationships (5), we get

$$(\Delta T_w/\Delta T)(1+\alpha/\varepsilon^2\beta) + (\alpha/\varepsilon^2)^{\frac{4}{3}}(\Delta T_w/\Delta T)^{\frac{4}{3}} \sim 1 \tag{6}$$

The relevant parameters β, α, and ε are defined as follows:

$$\beta = \frac{\lambda}{\lambda_c}\frac{1}{\sqrt{\pi}}\frac{[Ja/(\gamma Pr_c)]^{\frac{1}{4}}}{[H/(LPe)]^{\frac{1}{2}}}$$

$$\alpha = \frac{\lambda_w}{\lambda_c}\frac{h}{L}\left(\frac{Ja}{\gamma Pr_c}\right)^{\frac{1}{4}}, \qquad \varepsilon = \frac{h}{L} \tag{7}$$

where β is the ratio of the thermal resistance of the condensed fluid to the thermal resistance of the Darcy cooling flow. The parameter α relates the heat conducted by the wall to the heat convected from the condensed vapor fluid. Here ε is the aspect ratio of the wall, $\varepsilon = h/L$, which is assumed to be very small compared with unity. From relationships (2), (5), and (6), it can easily be shown that for values of β of the order unity and $\alpha/\varepsilon^2 \gg 1$

$$\delta_c/L \sim (Ja/\gamma Pr_c)^{\frac{1}{4}}, \qquad \Delta T_w/\Delta T \sim \varepsilon^2/\alpha \ll 1$$
$$\Delta T_c/\Delta T \sim 1, \qquad \Delta T_\infty/\Delta T \sim 1 \tag{8}$$

Thus, for values of α/ε^2 very large compared with unity, the transverse temperature variations at the wall are very small compared with the overall temperature difference. This limit corresponds to the thermally thin wall regime, and because of the practical applications, we develop analytical and numerical solutions for this regime in the following sections.

III. Governing Equations

The nondimensional governing equations for the wall and the condensed fluid are given next, together with the corresponding initial and boundary conditions. For this conjugated heat transfer problem, we add the thermal compatibility conditions. Finally, the energy equation for the Darcy cooling flow solved through of the Duhamel's theorem, is also presented. By the introduction of the nondimensional variables for the solid

$$\theta_w = (T_s - T_w)/(T_s - T_\infty), \qquad \chi = x/L, \qquad z = (y - H/2)/h \tag{9}$$

and for condensed flow,

$$u = \frac{\bar{u}}{\sqrt{gLJa/Pr_c}} = \Delta^2\frac{\partial f_c}{\partial \eta_c}$$

$$v = \frac{\bar{v}\gamma^{\frac{1}{4}}}{(Ja/Pr_c)^{\frac{3}{4}}\sqrt{gL}} = -\frac{\partial\left(\Delta^3 f_c\right)}{\partial\chi} + \Delta^2\eta_c\frac{\mathrm{d}\Delta}{\mathrm{d}\chi}\frac{\partial f_c}{\partial\eta_c}$$

$$\theta_c = \frac{T_s - T_c}{T_s - T_\infty}, \qquad \Delta = \frac{\delta_c(x)}{L[Ja/(\gamma Pr_c)]^{\frac{1}{4}}}$$

$$\eta_c = \frac{y - (H/2 + h)}{\delta_c(x)} \tag{10}$$

the nondimensional governing equations transform as follows:

$$\frac{\partial^2\theta_w}{\partial\chi^2} + \frac{1}{\varepsilon^2}\frac{\partial^2\theta_w}{\partial z^2} = 0 \tag{11}$$

$$\frac{\partial^3 f_c}{\partial\eta_c^3} + 1 = \frac{Ja}{Pr_c}\Delta^4\left\{\frac{\partial f_c}{\partial\eta_c}\frac{\partial^2 f_c}{\partial\chi\partial\eta_c} - \frac{\partial f_c}{\partial\chi}\frac{\partial f_c}{\partial\eta_c} + \frac{1}{\Delta}\frac{\mathrm{d}\Delta}{\mathrm{d}\chi}\left[2\left(\frac{\partial f_c}{\partial\eta_c}\right)^2 - 3f_c\frac{\partial^2 f_c}{\partial\eta_c^2}\right]\right\} \tag{12}$$

$$\frac{\partial^2\theta_c}{\partial\eta_c^2} = Ja\Delta^4\left(\frac{\partial f_c}{\partial\eta_c}\frac{\partial\theta_c}{\partial\chi} - \frac{\partial f_c}{\partial\chi}\frac{\partial\theta_c}{\partial\eta_c} - \frac{2}{\Delta}\frac{\mathrm{d}\Delta}{\mathrm{d}\chi}f_c\frac{\partial\theta_c}{\partial\eta_c}\right) \tag{13}$$

The momentum and energy equation for the condensed fluid were derived using the boundary-layer approximation. The adiabatic boundary conditions for the wall are given by

$$\left.\frac{\partial\theta_w}{\partial\chi}\right|_{\chi=0,1} = 0 \tag{14}$$

whereas the boundary conditions associated with the condensed fluid are given by

$$f_c(\chi,0) = \left.\frac{\partial f_c}{\partial\eta_c}\right|_{\eta_c=0} = 0, \qquad \theta_c(\chi,1) = \left.\frac{\partial^2 f_c}{\partial\eta_c^2}\right|_{\eta_c=1} = 0 \tag{15}$$

The last condition in Eq. (15) arises from the balance of tangential shear stress at the interface.[2] In the preceding Eqs. (12) and (13), the normalized nondimensional thickness of the condensed film, $\Delta(\chi)$, is unknown and must be obtained from the analysis. Thus, the energy balance at the condensed-vapor interface provides the spatial evolution of Δ as follows:

$$4\Delta\frac{\mathrm{d}\left[\Delta^3 f_c(\chi,1)\right]}{\mathrm{d}\chi} = -\left.\frac{\partial\theta_c}{\partial\eta_c}\right|_{\eta_c=1}, \quad \text{with} \quad \Delta(\chi=0)=0 \tag{16}$$

Furthermore, we also need the compatibility conditions at both surfaces of the wall. They are the continuity in the temperatures and the heat fluxes and may be written as follows.

External surface:

$$\theta_w(\chi,1) - \theta_c(\chi,0) = 0, \qquad \left.\frac{\partial\theta_w}{\partial z}\right|_{z=1} = \frac{\varepsilon^2}{\alpha}\frac{1}{\Delta}\left.\frac{\partial\theta_c}{\partial\eta_c}\right|_{\eta_c=0} \tag{17}$$

Internal surface:

$$\theta_w(\chi,1) - \theta(\chi,0) = 0, \qquad \left.\frac{\partial\theta_w}{\partial z}\right|_{z=0} = -\frac{\sqrt{\pi}\beta\varepsilon^2}{\alpha}\frac{1}{\chi^{\frac{1}{2}}}\left.\frac{\partial\theta}{\partial\zeta}\right|_{\zeta=0} \tag{18}$$

To analyze the Darcy cooling flow inside of the porous channel, we use the following nondimensional variables θ and ζ:

$$\theta = (T_s - T)/(T_s - T_\infty), \qquad \zeta = (HPe/4L)^{\frac{1}{2}}\left[(1 - 2y/H)\big/\chi^{\frac{1}{2}}\right] \tag{19}$$

where the last variable ζ was selected by applying the boundary-layer approximation for large values of the Péclet number $Pe \gg 1$. In this case, the longitudinal heat conduction terms of the energy equation for the Darcy cooling flow can be neglected. For $Pe \gg 1$, the thermal boundary layer is confined to a thin region of the fluid adjacent to the inner surfaces of the porous channel, where the order-of-magnitude energy balance of longitudinal convection and transverse conduction terms across the boundary layer are dominating. This energy balance justifies the use of the foregoing inner or stretching variable $\zeta = (HPe/4L)^{1/2}[(1-2y/H)/\chi]^{1/2} \sim \mathcal{O}(1)$. Therefore, the simplified energy equation for the porous media is given by

$$\frac{\partial^2\theta}{\partial\zeta^2} + \frac{\zeta}{2}\frac{\partial\theta}{\partial\zeta} = \chi\frac{\partial\theta}{\partial\chi} \tag{20}$$

together with the boundary conditions

$$\theta(\chi,0)-\theta_w(\chi,0)=0, \qquad \theta(\chi,\infty)-1=0 \tag{21}$$

The linear energy equation (20) can be easily integrated using Duhamel's theorem. (The details are omitted for simplicity.) For this case, the nondimensional heat flux at the inner surface of the wall can be written as follows[11]:

$$\left.\frac{\partial\theta}{\partial\zeta}\right|_{\zeta=0}=\frac{1}{\sqrt{\pi}}\left(1-\theta_{wl}-\int_0^{\chi}K(\chi,\chi')\frac{d\theta'_w}{d\chi'}\,d\chi'\right) \tag{22}$$

where the kernel K for the present laminar Darcy flow is given by $K(\chi,\chi')=(1-\chi'/\chi)^{-1/2}$ and θ_{wl} is the value of the nondimensional temperature at the upward end of the wall.

IV. Thermally Thin Wall Limit ($\alpha/\varepsilon^2\gg 1$)

In this regime, the nondimensional temperature of the wall depends only on the χ coordinate in a first approximation, as predicted in relationship (8). Therefore, Eq. (11) can be integrated along the transverse coordinate and, after applying the compatibility conditions (16) and (17) at both vertical surfaces of the wall, together with Eq. (22), we obtain

$$\alpha\frac{d^2\theta_w}{d\chi^2}+\frac{1}{\Delta}\left.\frac{\partial\theta_c}{\partial\eta_c}\right|_{\eta_c=0}=\frac{\beta}{\chi^{\frac{1}{2}}}\left[\theta_{wl}-1+\int_0^{\chi}K(\chi,\chi')\frac{d\theta'_w}{d\chi'}\,d\chi'\right] \tag{23}$$

The second term on the left-hand side of Eq. (23) denotes the heat transferred from the condensed fluid, and the term on the right-hand side is the heat transferred to the porous media flow. The boundary conditions at the wall ends transform to

$$\left.\frac{d\theta_w}{d\chi}\right|_{\chi=0,1}=0 \tag{24}$$

For values of $\beta\gg 1$, the nondimensional wall temperature is practically 1, and the laminar-film-condensation process corresponds to Nusselt's limit to a first approximation. A uniform temperature of the wall also is obtained for large values of α, where the temperature profiles depend on the values of β. The limiting case of $\beta=0$ represents a simple conjugated heat transfer process between the wall and the porous media flow, without any condensation phenomena. The system of Eqs. (2), (13), (19), and (23), together with the given initial and boundary conditions, contains three differential equations and one integro–differential equation, respectively, for the unknowns $f_c(\chi,\eta_c)$, $\theta_c(\chi,\eta_c)$, $\Delta(\chi)$, and $\theta_w(\chi)$ with five different nondimensional parameters, Ja, Pr_c, α, β, and ε. In the following sections, we analyze the limit characterized by small Jacob number $Ja\to 0$, with the Prandtl number of the condensed phase of order unity. The limits of large and small values of α and β of order unity for the thermally thin wall regime ($\alpha/\varepsilon^2\gg 1$) are considered.

V. Asymptotic Solution for $Ja\to 0$

In this limit, and assuming Prandtl number Pr_c to be of the order unity, the solution of the governing equations for the condensed-phase equations (12) and (13) with Eq. (15) is given by

$$\theta_c=\theta_w(\chi)(1-\eta_c), \qquad f_c(\eta_c)=\tfrac{1}{2}\eta_c^2[1-(\eta_c/3)] \tag{25}$$

The appropriate or reduced Nusselt number Nu_c for this problem is given by

$$Nu_c=\frac{qL}{\lambda_c(T_s-T_\infty)}\left(\frac{Ja}{\gamma Pr_c}\right)^{\frac{1}{4}}=-\frac{1}{\Delta}\left.\frac{\partial\theta_c}{\partial\eta_c}\right|_{\eta_c=0}=\frac{\theta_w}{\Delta} \tag{26}$$

Thus, the nondimensional energy balance equation (19) at the interface vapor–condensed fluid transforms to

$$\frac{d\Delta^4}{d\chi}=\theta_w \tag{27}$$

and Eq. (23) for the solid can now be rewritten as follows:

$$\alpha\frac{d^2\theta_w}{d\chi^2}-\frac{\theta_w}{\Delta}=\frac{\beta}{\chi^{\frac{1}{2}}}\left[\theta_{wl}-1+\int_0^{\chi}K(\chi,\chi')\frac{d\theta'_w}{d\chi'}\,d\chi'\right] \tag{28}$$

In this limit, the solution depends on two free parameters: α and β. The nonlinear system of Eqs. (27) and (28) must be solved with the initial condition, $\Delta(0)=0$, and the adiabatic conditions at the ends of the wall given by Eqs. (24). We use numerical techniques[8] to solve these equations, and we have explored asymptotic solutions for large and small values of the parameter α.

A. Solution for $\alpha\gg 1$

In this limit, the solution is regular and the nondimensional temperature of the wall, θ_w, changes very little (of order of α^{-1}) in the longitudinal direction. We assume that the nondimensional temperature of the wall, as well as the nondimensional condensed-layer thickness, can be expanded as follows:

$$\theta_w(\chi)=\theta_0(\chi)+\sum_{j=1}^{\infty}\alpha^{-j}\theta_j(\chi)$$

$$\Delta(\chi)=\Delta_0(\chi)+\sum_{j=1}^{\infty}\alpha^{-j}\Delta_j(\chi) \tag{29}$$

Introducing these relationships into Eqs. (27) and (28), we obtain, after collecting terms of the same power of α, the following sets of equations:

$$\frac{d^2\theta_0}{d\chi^2}=0, \qquad \frac{d\Delta_0^4}{d\chi}=\theta_0 \tag{30}$$

$$\frac{d^2\theta_1}{d\chi^2}=\frac{\theta_0}{\Delta_0}+\frac{\beta}{\chi^{\frac{1}{2}}}(\theta_0-1), \qquad \frac{4d\left(\Delta_0^3\Delta_1\right)}{d\chi}=\theta_1 \tag{31}$$

$$\frac{d^2\theta_2}{d\chi^2}=\frac{\theta_0}{\Delta_0}\left(\frac{\theta_1}{\theta_0}-\frac{\Delta_1}{\Delta_0}\right)+\frac{\beta}{\chi^{\frac{1}{2}}}\left(\theta_{1l}+\int_{\theta_{1l}}^{\theta_1}K(\chi,\chi')\,d\theta'_1\right) \tag{32}$$

$$\frac{d\left(4\Delta_0^3\Delta_2+6\Delta_0^2\Delta_1^2\right)}{d\chi}=\theta_2 \tag{33}$$

etc., with the following initial and boundary conditions for all $i\geq 0$:

$$\Delta_i(0)=0, \qquad \left.\frac{d\theta_i}{d\chi}\right|_{\chi=0,1}=0 \tag{34}$$

Integration of Eqs. (30), with the corresponding initial and boundary conditions (34), gives $\theta_0=C_0$ and $\Delta_0=C_0^{1/4}\chi^{1/4}$, with the constant $C_0(\beta)$ to be obtained in the next order. We can integrate Eq. (31), and on applying the adiabatic boundary conditions at both ends of the wall, we obtain the leading order of the nondimensional wall temperature as a function of the parameter β in the form

$$\beta=\tfrac{2}{3}\left[C_0^{\frac{3}{4}}\big/(1-C_0)\right] \tag{35}$$

In particular, the leading order of the nondimensional thickness of the condensate at the lower end of the wall, $\Delta_0(1)$, which is a measure of the condensation efficiency, after employing the limits of large and small values of β compared with unity, is given by

$$\Delta_0(1)\sim\left(\tfrac{3}{2}\right)^{\frac{1}{3}}\beta^{\frac{1}{3}}-\tfrac{1}{3}\left(\tfrac{3}{2}\right)^{\frac{5}{3}}\beta^{\frac{5}{3}} \tag{36}$$

for $\beta\ll 1$ and

$$\Delta_0(1)\sim 1-1/6\beta+1/24\beta^2 \tag{37}$$

for $\beta\gg 1$.

Introducing the solutions $\theta_0 = C_0$ and $\Delta_0 = C_0^{1/4}\chi^{1/4}$ into Eq. (31), and integrating this equation twice, after considering the appropriate initial and boundary conditions, we obtain

$$\theta_1 = (8/3)C_0^{\frac{3}{4}}\left(C_1 - (1/3)\chi^{\frac{3}{2}} + (2/7)\chi^{\frac{7}{4}}\right) \tag{38}$$

where C_1 is an integration constant related to the temperature of the wall at the top end and must be determined by solving the next-higher-order equation. Thus, the first-order correction to the nondimensional condensed-layer thickness gives

$$\Delta_1 = (2/3)\chi^{\frac{1}{4}}\left[C_1 - (2/15)\chi^{\frac{3}{2}} + (8/77)\chi^{\frac{7}{4}}\right] \tag{39}$$

Integration of Eq. (32) gives the value of C_1, after applying the adiabatic boundary conditions at both ends, as follows:

$$C_1(\beta) = \frac{0.0294 + 0.0732\beta C_0^{\frac{1}{4}}}{1 + 2\beta C_0^{\frac{1}{4}}} \tag{40}$$

Therefore, up to the first order, the condensed-layer thickness is given by

$$\Delta = \chi^{\frac{1}{4}}\left\{C_0^{\frac{1}{4}} + (2/3\alpha)\left[C_1 - (2/15)\chi^{\frac{3}{2}} + (8/77)\chi^{\frac{7}{4}}\right]\right\} + \mathcal{O}(\alpha^{-2}) \tag{41}$$

the nondimensional wall temperature is given by

$$\theta_w = C_0 + (8/3\alpha)C_0^{\frac{3}{4}}\left[C_1 - (1/3)\chi^{\frac{3}{2}} + (2/7)\chi^{\frac{7}{4}}\right] + \mathcal{O}(\alpha^{-2}) \tag{42}$$

and the reduced Nusselt number now takes the form

$$Nu_c^* = \left(C_0^{\frac{3}{4}}\big/\chi^{\frac{1}{4}}\right)\left\{1 + \left(2\big/C_0^{\frac{1}{4}}\alpha\right)\left[C_1 - (2/5)\chi^{\frac{3}{2}} + (80/231)\chi^{\frac{7}{4}}\right]\right\} + \mathcal{O}(\alpha^{-2}) \tag{43}$$

The leading term on the right-hand side of the preceding equations represents the classical Nusselt solution[1] for an isothermal wall.

B. Solution for $\alpha \to 0$

For small values of α compared with unity, the longitudinal heat conduction term in Eq. (28) can be neglected. For values of $\alpha \to 0$, but large compared with ε^2, the thermally thin wall approximation is still valid. This case represents a singular limit due to existence of two longitudinal heat conducting layers close at both ends to satisfy the adiabatic boundary conditions. The structure of these thermal boundary layers is not presented here because they only have a local thermal influence. Outside of these inner zones, longitudinal heat conduction through the wall is negligible, reducing the governing equations up to the leading order to the following set:

$$\frac{\theta_{we}}{\Delta_e} = -\frac{\beta}{\chi^{\frac{1}{2}}}\left[\int_0^{\chi} K(\chi,\chi')\frac{\mathrm{d}\theta'_{we}}{\mathrm{d}\chi'}\,\mathrm{d}\chi'\right] \tag{44}$$

$$\frac{\mathrm{d}\Delta_e^4}{\mathrm{d}\chi} = \theta_{we} \tag{45}$$

Where the subscript e represents the outer nonconducting zone. To solve the preceding system, we need the initial conditions to be obtained from matching with the inner heat conducting zone. This nonlinear integral equation must be solved numerically.[8] Equations (44) and (45) can be transformed to a parameter-free system by introducing the following variables

$$\psi = \Delta_e^4\big/\beta^4, \qquad \sigma = \chi/\beta^4 \tag{46}$$

giving

$$\frac{\theta_{we}}{\psi^{\frac{1}{4}}} = -\frac{1}{\sigma^{\frac{1}{2}}}\int_0^{\sigma} K(\sigma,\sigma')\frac{\mathrm{d}\theta'_{we}}{\mathrm{d}\sigma'}\,\mathrm{d}\sigma' \tag{47}$$

$$\frac{\mathrm{d}\psi}{\mathrm{d}\sigma} = \theta_{we} \tag{48}$$

The asymptotic solution, after applying the initial conditions, is given by

$$\theta_{we} = 1 - \left[\frac{4\Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{4}\right)}\right]\frac{\chi^{\frac{1}{4}}}{\beta} + \left[\frac{32}{5\pi}\frac{\Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{4}\right)}\right]\frac{\chi^{\frac{1}{2}}}{\beta^2} + \mathcal{O}(\beta^{-3}) \tag{49}$$

$$\Delta_e = \chi^{\frac{1}{4}} - a_1\frac{\chi^{\frac{1}{2}}}{\beta} + a_2\frac{\chi^{\frac{3}{4}}}{\beta^2} + \mathcal{O}(\beta^{-3}) \tag{50}$$

where the constants a_1 and a_2 are given by

$$a_1 = \frac{4\Gamma\left(\frac{3}{4}\right)}{5\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{4}\right)}, \qquad a_2 = \frac{8\Gamma\left(\frac{3}{4}\right)}{5\pi\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{4}\right)}\left[1 - \frac{9\pi\Gamma\left(\frac{3}{4}\right)}{10\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{4}\right)}\right]$$

The leading terms in the preceding Eqs. (49) and (50) are exactly the same as for the case of $\alpha \to \infty$, indicating that for $\beta \to \infty$ there is no influence of the longitudinal heat conduction in the wall on the condensation process. In this case, the temperature of the wall is very close to T_∞. Similarly, for small values of β (large values of σ), the integral term of Eq. (44) reduces in a first approximation to

$$\int_0^{\sigma} K(\sigma,\sigma')\frac{\mathrm{d}\theta'_{we}}{\mathrm{d}\sigma'}\,\mathrm{d}\sigma' \sim -1 \tag{51}$$

for $\sigma \gg 1$. Therefore, the asymptotic solution of Eqs. (47) and (48) is then given by

$$\psi \sim \left(\tfrac{3}{2}\right)^{\frac{4}{3}}\sigma^{\frac{2}{3}}, \qquad \theta_{we} \sim \left(\tfrac{3}{2}\right)^{\frac{1}{3}}\sigma^{-\frac{1}{3}} \tag{52}$$

for $\sigma \gg 1$. Finally, the nondimensional condensed-layer thickness and temperature of the wall at the downward end are given by

$$\Delta_{eL} = \Delta_e(\chi = 1) \sim \left(\tfrac{3}{2}\right)^{\frac{4}{3}}\beta^{\frac{4}{3}}$$

$$\theta_{weL} = \theta_{we}(\chi = 1) \sim \left(\tfrac{3}{2}\right)^{\frac{1}{3}}\beta^{\frac{4}{3}} \tag{53}$$

for $\beta \to 0$.

VI. Results

In the limit of very large values of β, the temperature of the wall is practically uniform and close to the value of the temperature of the cooling flow. This is the well-known Nusselt solution. Here, the longitudinal heat conduction does not play an important role, even for large values of α. On the other hand, for finite values of β and $\alpha \to 0$, the system of Eqs. (27) and (28) is singular. This means that it is necessary to include the inner heat conducting layers at both ends of the wall. However, these thermal conduction boundary layers have only a local influence, and we do not present the solution in these layers because it will increase unnecessary the length of the paper. Outside of these inner regions, there is an outer zone where the longitudinal heat conduction through the wall is negligible in a first approximation. In this outer zone and for $\alpha = 0$, the nondimensional temperature of the heat conducting wall and the thickness of the condensate film (at $\chi = 1$) as a function of the parameter β are shown in Figs. 2 and 3, by use of asymptotic and numerical techniques. It is clear from Fig. 2 that, for increasing values of the parameter β, the dimensional temperature of the wall decreases because the thermal resistance of the condensed fluid becomes larger than the corresponding resistance of the cooling flow. In Fig. 3, the nondimensional thickness of the film is shown for increasing values of the parameter β. The asymptotic solutions in the limit $\alpha = 0$, for $\beta \to 0$ and $\beta \to \infty$, are also plotted, together with the leading term for $\alpha \to \infty$. The influence of the longitudinal heat conduction through the wall is very small as we can see in Fig. 3. However, we obtain slightly more condensation at the lower end of the wall as the value of α decreases as predicted by the first-order correction given

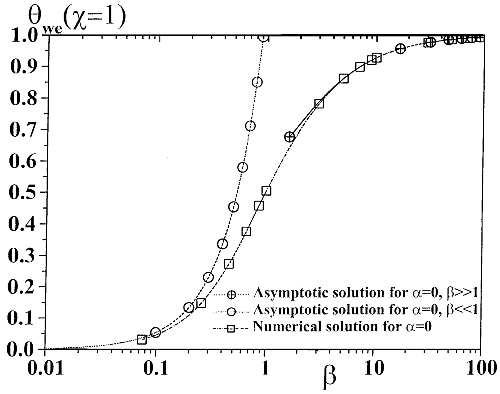

Fig. 2 Nondimensional wall temperature at the downward end as a function of β, for $\alpha = 0$ and asymptotic solutions for large and small values of β.

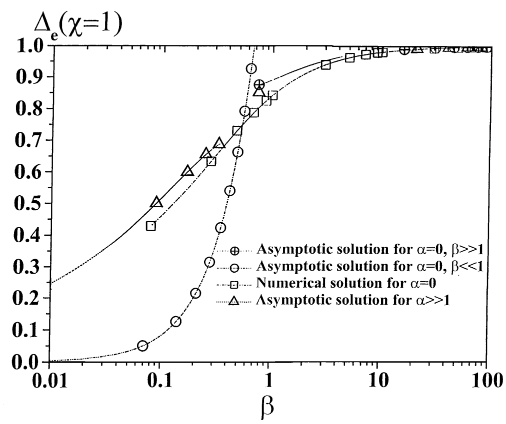

Fig. 3 Nondimensional condensed layer thickness at the downward end as a function of β, for $\alpha = 0$, asymptotic solutions for large and small values of β, and asymptotic solution for $\alpha \gg 1$.

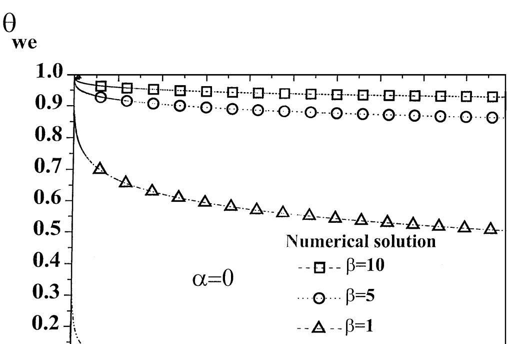

Fig. 4 Numerical solution for nondimensional wall temperature profile as a function of χ for different values of β and $\alpha = 0$.

by Eq. (39), where $\Delta_1(1)$ is positive for all values of β. For large values of β, the asymptotic solutions give for $\alpha \to \infty$,

$$\Delta(1) \sim 1 - 0.16666/\beta + 0.04166/\beta^2 + \cdots \tag{54}$$

and for $\alpha = 0$,

$$\Delta(1) = 1 - 0.15255/\beta + 0.04475/\beta^2 + \cdots \tag{55}$$

Figure 4 shows the numerical results of the nondimensional temperature of the wall as a function of χ for $\alpha = 0$ and different values of β. The nondimensional temperature of the wall always decreases along the wall. However, this effect is even more important for small values of β. The results also give the Nusselt classical solution as a particular case. For $\alpha = 0$, Fig. 5 shows the nondimensional condensed-layer thickness as a function of χ, for different values of β. We can see that the nondimensional thickness of the condensed fluid always increases with increasing values of β, and this asymptotically attains the Nusselt solution for $\beta \to \infty$.

For $\alpha \sim 1$ and finite values of β, the problem defined by Eqs. (27) and (28) must be solved numerically. Figure 6 shows the corresponding numerical and asymptotic solutions for the nondimensional temperature as a function of the longitudinal coordinate χ, for different values of β and $\alpha = 5$. The temperature profiles in the wall tend to be uniform. The asymptotic solution for large values of α introduces a 2% error for values of α around unity. Similarly,

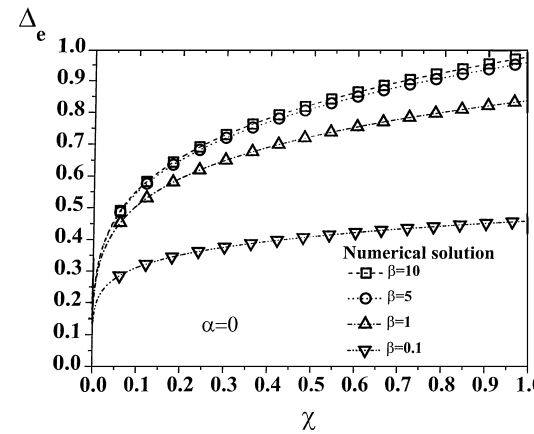

Fig. 5 Numerical solution for nondimensional condensed-layer thickness as a function of χ for different values of β and $\alpha = 0$.

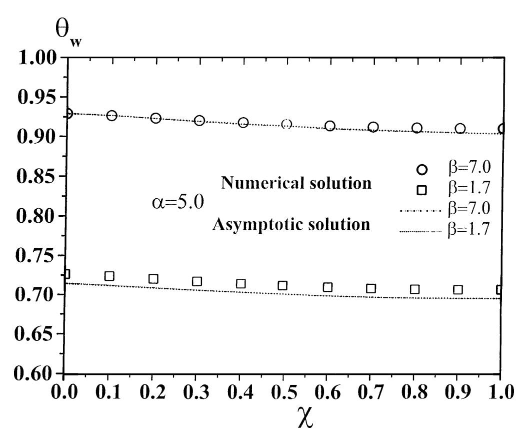

Fig. 6 Numerical and asymptotic solutions for the nondimensional wall temperature profile as a function of χ for different values of β and $\alpha = 5.0$.

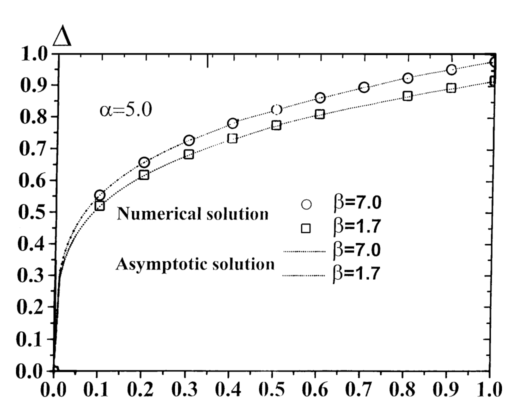

Fig. 7 Nondimensional condensed-layer thickness as a function of χ for different values of β and $\alpha = 5.0$.

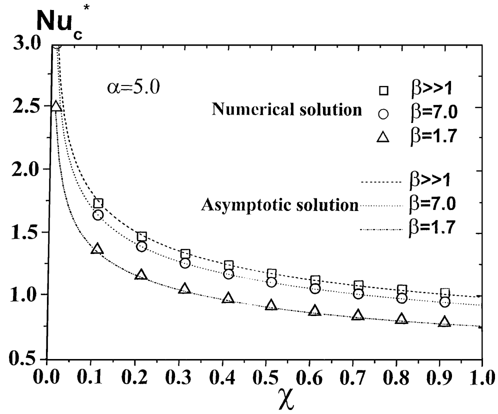

Fig. 8 Reduced Nusselt number for the condensed fluid for different values of β and $\alpha = 5.0$.

Fig. 7 shows the results for the nondimensional thickness of the condensed layer as a function of χ for different values of β and $\alpha = 5$. Figure 7 shows the big influence of parameter β. For comparison, the two-term asymptotic solution obtained for large values of α gives a reasonable agreement even for values of α of order unity. Finally, in Fig. 8 the influence of the cooling mechanism controlled by parameter β shows that, for increasing values of this parameter, the corresponding Nusselt number is also increased. Also, in Fig. 8, the comparison between both solutions shows a good agreement for the reduced local Nusselt number with variable temperature through the wall, reflected through the parameter $\alpha = 5$.

VII. Conclusions

The laminar-film-condensation process of a saturated vapor in contact with the external surfaces of a porous channel has been analyzed for small values of the Jacob number, using asymptotic as well as numerical techniques. The finite thermal conductivity of the wall material allows to transfer heat by conduction upstream through the wall. The local heat forced convection through the inner lateral surfaces of the walls, affected strongly by the axial heat conduction, governs the spatial evolution of the wall temperature and the condensed-layer thickness. Although, there is a large influence of the longitudinal conduction effects through the walls on the temperature distribution, there is only a slight influence on the mass flow rate of condensate at the lower edge of the wall. Decreasing the longitudinal heat conductance α produces a slight increase in the global condensation rate. This is true for the thermally thin wall regime, that is, for values of α very large compared with ε^2. In the thermally thick wall regime ($\alpha/\varepsilon^2 \sim 1$), it reverts, producing a decreasing condensation rate for smaller values of the parameter α. The latter regime is not considered here.

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